

Generalized Skymions and hairy black holes in asymptotically AdS_4 spacetime

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We investigate the properties of spherically symmetric black hole solutions in the generalized Einstein-Skyrme model theory in four-dimensional asymptotically anti-de Sitter space-time. The dependencies of the Skymion fields on the cosmological constant and on the strength of the effective gravitational coupling are examined. We show that the increase of the absolute value of the cosmological constant qualitatively yields the same effect as increasing of the effective gravitational coupling. We confirm that, similar to the model in the asymptotically flat space-time, a necessary condition for the existence of black holes with Skymionic hair is the inclusion of the Skyrme term.

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I. INTRODUCTION

Existence of black holes appear to be an unavoidable consequence of General Relativity as well as its various extensions. Since ground-breaking work by Hawking [1], there has been an increasing interest in investigation of various aspects of these solutions, especially the thermodynamics of the black holes.

When gravity is coupled to the matter field, the complete system of the Einstein equations may support both the existence of the gravitating spacially localised particle-like solutions and black holes. The soliton solutions may be linked to the black holes with hair, which represent gravitating field configurations possessing both an event horizon and a non trivial structure of the matter fields (see, e.g. [2]).

The first example of the hairy black hole solutions was constructed in the asymptotically flat Einstein-Skyrme model [3, 4]. These solutions are stable, asymptotically flat and possess a regular horizon, furthermore, they may be viewed as bound states of Skymions and Schwarzschild black holes [5].

The corresponding globally regular gravitating Skymions in asymptotically flat space were considered in many works, in particular it was shown [4, 6] that there are two branches of solutions. One of these braches is linked to the usual flat space Skymion solution, while another, higher in energy branch, in the limit of vanishing effective coupling constant approaches the Bartnik-McKinnon solution of the $SU(2)$ Einstein-Yang-Mills theory [7].

The stability analysis shows [6, 8] that the lowest in energy branch is stable with respect to linear perturbations, while the upper branch is unstable, it may collapse to the Schwarzschild black hole.

Various examples of the gravitating Skyrmions and hairy black holes were investigated over last 20 years. Axially symmetric globally regular Skyrmions and black holes with hair were constructed in [9], self-gravitating multi-Skyrmion configurations with discrete symmetry were investigated in [10], spinning gravitating Skyrmions were studied in [11]. Recently self-gravitating axially symmetric sphaleron solutions of the Einstein-Skyrme model were constructed [12].

An interesting development is related with investigation of the black holes with Skyrmion hair in anti-de Sitter (AdS) spacetime, in particular in the context of the AdS/CFT conjecture [13, 14]. On the other hand, the Skyrmion-like solutions arise as configurations, which are holographically dual to a D4-brane wrapped on the non-trivial four-cycle in the (4+1)-dimensional bulk spacetime with AdS geometry and conformal boundary [15].

It is known that the hairy black holes in the asymptotically AdS spacetime can be stabilized as the cosmological constant Λ grows [18]. Indeed, the no-hair conjecture was revisited for the scalar theory in asymptotically AdS spacetime [16], it was confirmed that the spherically symmetric stable black hole solutions with scalar hair can exist in this model. Further, the Einstein-Skyrme model in the asymptotically AdS spacetime was considered in [17], it was shown that the stable black holes with Skyrmion hair exist up for some maximum value of $|\Lambda|$. Similar to the case of the solitons of the Einstein-Skyrme model in asymptotically flat spacetime, there are two branches of solutions for each value of Λ as well as the effective gravitational coupling constant.

Over the last decade a number of modifications of the Skyrme model have been proposed. The main motivation was to improve phenomenological predictions of the Skyrme model [19], which was originally suggested as a candidate for low-energy QCD effective theory. In this framework the nuclei are considered as topological solitons. The Skyrme Lagrangian may be recovered from the $1/N$ expansion in derivatives of non-perturbative low-energy QCD effective action, then one has to take into account the first two terms only. However, this restriction is not very natural, thus it was suggested to generalize the original Skyrme model by inclusion into Lagrangian some additional terms which are higher order either in derivatives [20–23]. The natural extension of the basic Skyrme model is to include the sextic term, which is allowed by requirements of Poincare invariance and existence of the usual Hamiltonian formulation [20]. This term is proportional to the square of the topological current, thus it is quadratic in time derivatives, like the quartic Skyrme term it allows for an usual time dynamics of the system. It was also noted that inclusion of this term may provide better match with the experimental data of nuclei [24]. Furthermore, the extended Skyrme model with sextic term can be reduced to the BPS submodel [25, 26], which is integrable.

The black hole solutions in the general Skyrme model were studied recently in the asymptotically flat spacetime [28, 29], the regular gravitating Skyrmions of the BPS submodel were

considered in fixed AdS spacetime without backreaction [30]. An interesting observation is that the presence of the quartic Skyrme term is a necessary condition for the existence of black holes with Skyrmonic hair, there is no hairy black holes in the BPS submodel in the asymptotically flat spacetime. However, to our best knowledge, similar study in asymptotically AdS spacetime has not been reported so far.

The main purpose of this work is to extend the consideration of papers [28, 29] to the case of the regular self-gravitating solutions of the general Skyrme model and the corresponding static hairy black hole solutions in asymptotically AdS₄ spacetime.

The paper is organized as follows: in the next section we discuss the generalized Skyrme model in asymptotically AdS₄ space-time, the spherically symmetric ansatz which we apply to parameterize the action of the Einstein-Skyrme theory, the boundary conditions imposed to get regular solution and establish the equations. The numerical results for the case of regular self-gravitating Skyrmons are presented in Section 3 where we compare the results to the numerical solutions of the Skyrme model in asymptotically flat space-time. The properties of the hairy black holes are discussed in the Section IV. We give our conclusions and remarks in the final section.

II. GENERALIZED EINSTEIN-SKYRME-ADS MODEL

The general Skyrme model is a Poincare invariant, nonlinear sigma model field theory. In a curved spacetime with metric $g_{\mu\nu}$ the Lagrangian of the model takes the form

$$\mathcal{L} \equiv a\mathcal{L}_2 + b\mathcal{L}_4 + c\mathcal{L}_6 + \mathcal{L}_0, \quad (1)$$

where the quadratic and quartic in derivative terms

$$\mathcal{L}_2 = \frac{1}{8}g^{\mu\nu}\text{Tr}(L_\mu L_\nu), \quad \mathcal{L}_4 = \frac{1}{64}g^{\mu\nu}g^{\rho\sigma}\text{Tr}([L_\mu, L_\rho][L_\nu, L_\sigma]), \quad (2)$$

are the sigma model and Skyrme parts, respectively. The parameters a, b and c are dimensionfull non-negative coupling constants. The model is expressed in terms of the $su(2)$ -valued left chiral current $L_\mu = U^\dagger \partial_\mu U$, associated with the $SU(2)$ -valued Skyrme field $U = \sigma \cdot \mathbb{I} + i\pi^a \cdot \tau^a$. The quartet of the fields (σ, π^a) is restricted to the surface of the unit sphere, $\sigma^2 + \pi^a \cdot \pi^a = 1$. The topological boundary condition on the Skyrme field is that $U \rightarrow \mathbb{I}$ as $r \rightarrow \infty$, thus the field is a map $U : S^3 \rightarrow S^3$, which belongs to an equivalence class characterized by the homotopy group $\pi_3(S^3)$. The corresponding topological current is

$$\mathcal{B}^\mu = -\frac{1}{24\pi^2\sqrt{-g}}\epsilon^{\mu\nu\rho\sigma}\text{Tr}(L_\nu L_\rho L_\sigma). \quad (3)$$

The topological charge corresponds to zeroth component of this current, thus the winding number is $B = \int d^3x \sqrt{-g} \mathcal{B}^0$.

Further, the extended Skyrme model (1) includes a sextic term, which is given by the square of the topological current

$$\mathcal{L}_6 = \pi^4 \mathcal{B}_\mu \mathcal{B}^\mu. \quad (4)$$

It was noted that effectively this term corresponds to an inclusion of the repulsive interaction mediated by ω meson [24] and gives the leading contribution to the equation of state at high pressure or density [31].

The model also includes the potential term \mathcal{L}_0 , its form can be arbitrary. Here we will use the standard potential

$$\mathcal{L}_0 = \frac{m_\pi^2}{8} \text{Tr} (U - \mathbb{I}), \quad (5)$$

where the parameter m_π is the pion mass [41].

Note that in the flat space limit the model (1) can be reduced to the usual Skyrme model in the chiral limit by setting $c = m_\pi = 0$. On the other hand, setting $a = b = 0$ reduces the general model to the BPS submodel, with only the sextic term \mathcal{L}_6 and a potential term \mathcal{L}_0 . The solitons of this model saturate the topological bound, they have zero binding energy and, with a usual choice of the potential term (5), they are compactons [25].

Let us consider the extended Einstein-Skyrme model with a cosmological constant $\Lambda = -3/L^2$. The action is

$$S = \int d^4x \sqrt{-g} \left(\frac{R - 2\Lambda}{16\pi G} + \mathcal{L} \right). \quad (6)$$

where \mathcal{L} is the general Skyrme Lagrangian (1). The asymptotically flat limit $\Lambda = 0$ of this model was investigated in [28, 29]. The other limiting case $c = 0$ corresponds to the standard Skyrme model in the AdS spacetime [17].

Note that we can introduce the dimensionless radial coordinate $\tilde{r} = \sqrt{\frac{a}{b}} r$ and the effective gravitational coupling constant $\alpha^2 = 4\pi G a$. Then only four parameters effectively remain in the rescaled action:

$$a \rightarrow 1, \quad b \rightarrow 1, \quad c \rightarrow \tilde{c} = \frac{ac}{b^2}, \quad m_\pi^2 \rightarrow \tilde{m}_\pi^2 = m_\pi^2 \frac{b}{a^2}, \quad \Lambda \rightarrow \tilde{\Lambda} = \Lambda \frac{b}{a}. \quad (7)$$

Alternatively, one can consider the AdS length scale setting $r \rightarrow r\sqrt{\Lambda}$.

The Einstein equations can be obtained from the variation of the action (6) with respect to the metric,

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G T_{\mu\nu}, \quad (8)$$

where the Skyrme stress-energy tensor is defined as

$$\begin{aligned} T_{\mu\nu} &= \frac{2}{\sqrt{-g}} \frac{\delta(\sqrt{-g}\mathcal{L})}{\delta g^{\mu\nu}} \\ &= \frac{a}{4} \text{Tr} \left(L_\mu L_\nu - \frac{1}{2} g_{\mu\nu} L_\rho L^\rho \right) + \frac{b}{16} \text{Tr} \left([L_\mu, L_\rho] [L_\nu, L^\rho] - \frac{1}{4} g^{\mu\nu} [L_\rho, L_\sigma] [L^\rho, L^\sigma] \right) \\ &\quad + \frac{c}{64} \left(\text{Tr} (L_\mu [L_\alpha, L_\sigma]) \text{Tr} (L_\nu [L^\alpha, L^\sigma]) - \frac{1}{6} g_{\mu\nu} \text{Tr} (L_\alpha [L_\beta, L_\sigma]) \text{Tr} (L^\alpha [L^\beta, L^\sigma]) \right) \end{aligned} \quad (9)$$

Apart from (8), one should also consider the matter field equations, which are found from the variation of the action with respect to the Skyrme field U .

Hereafter we restrict our consideration to the static spherically symmetric configurations in the sector $B = 1$. Then the Skyrme field can be parametrized by the rotationally invariant hedgehog ansatz

$$U(\mathbf{r}) = e^{if(r)\hat{r}^a \cdot \tau^a} = \cos f(r) + i \sin f(r) \hat{r}^a \cdot \tau^a \quad (10)$$

where the Skymion profile function $f(r)$ is a real function of the radial variable with the topological boundary conditions $f(0) = \pi$, $f(\infty) = 0$, and \hat{r}^a the usual unit vector in \mathbb{R}^3 . We also make use of the static, spherically symmetric asymptotically AdS metric

$$ds^2 = \sigma^2(r)N(r)dr^2 - \frac{dr^2}{N(r)} - r^2(d\theta^2 + \sin^2\theta d\phi^2) \quad (11)$$

with the metric function $N(r), \sigma(r)$ depending only on r . It is convenient to define

$$N(r) = 1 - \frac{2M(r)}{r} - \frac{\Lambda r^2}{3}, \quad (12)$$

then in the limit $r \rightarrow \infty$ the metric function $M(r)$ approaches the so called Arnowitt-Deser-Misner (ADM) mass of the solution.

Substituting (10),(11) into (6) and varying it with respect to $N(r)$, $\sigma(r)$ and $f(r)$ we obtain the following system of coupled equations:

$$\begin{aligned} N_r &= -\frac{\alpha^2}{2} \left[a \left(N_r f_r^2 + \frac{2 \sin^2 f}{r} \right) + b \left(\frac{2 N f_r^2 \sin^2 f}{r} + \frac{\sin^4 f}{r^3} \right) + \frac{c N f_r^2 \sin^4 f}{r^3} + \frac{1}{2} m_\pi^2 r \sin^2 \frac{f}{2} \right] \\ &\quad + \frac{1-N}{r} - \Lambda r, \\ \sigma_r &= \frac{1}{2} \alpha^2 \sigma f_r^2 \left(ar + \frac{2b \sin^2 f}{r} + \frac{c \sin^4 f}{r^3} \right), \\ f_{rr} &= \frac{1}{a + \frac{2b \sin^2 f}{r^2} + \frac{c \sin^4 f}{r^4}} \left\{ a \left[\frac{\sin 2f}{N r^2} - f_r \left(\frac{N_r}{N} + \frac{\sigma_r}{\sigma} + \frac{2}{r} \right) \right] \right. \\ &\quad + b \left(\frac{2 \sin^3 f \cos f}{N r^4} + f_r \left(-\frac{2 \sin^2 f}{r^2} \frac{N_r}{N} - \frac{2 \sin^2 f}{r^2} \frac{\sigma_r}{\sigma} \right) - \frac{f_r^2 \sin 2f}{r^2} \right) \\ &\quad \left. + c \left(f_r \left(-\frac{\sin^4 f}{r^4} \frac{N_r}{N} + \frac{2 \sin^4 f}{r^5} - \frac{\sin^4 f}{r^4} \frac{\sigma_r}{\sigma} \right) - \frac{2 f_r^2 \sin^3 f \cos f}{r^4} \right) + \frac{m_\pi^2 \sin f}{2N} \right\}. \end{aligned} \quad (13)$$

Similar to the case of the general Skyrme model in asymptotically flat space [28], the system of equations (13) is invariant with respect to the scaling transformation $\sigma \rightarrow \lambda \sigma$. Thus, the function σ can be rescaled to set $\sigma \rightarrow 1$ as $r \rightarrow \infty$, furthermore this function can be eliminated from the equations completely since the third of these equations depends only on ratio $\frac{\sigma_r}{\sigma}$, given by the second equation. Thus,

$$\sigma(r) = \sigma(0) \exp \left[\frac{\alpha^2}{2} \int_0^r f_r^2 \left(ar + \frac{2b \sin^2 f}{r} + \frac{c \sin^4 f}{r^3} \right) dr \right], \quad (14)$$

where $\sigma(0)$ is an integration constant. However, it is convenient to keep the metric function σ in the numerical calculations.

The equations (13) can be solved numerically when we impose the appropriate boundary conditions. In the case of regular self-gravitating solitons, the boundary conditions at the origin and on the spacial asymptotic follows from the conditions of finiteness of the energy functional and topology of the Skyrme field in the sector of degree one:

$$f(0) = \pi, \quad N(0) = 1, \quad f(\infty) = 0, \quad \sigma(\infty) = 1. \quad (15)$$

III. GRAVITATING SOLITONS IN THE GENERALIZED EINSTEIN-SKYRME-ADS MODEL

Solutions of the system (13) are constructed numerically using the 8th order shooting algorithm based on the Dormand-Prince modification of the Runge-Kutta method [32]. They are also verified by boundary-value problem solving routines of open source package bvpSolve [33], the relative error is lower than 10^{-9} .

First, we consider the regular solutions of the system (13). The system of equations (13) depends on six parameters: four coupling constants in the matter fields sector, a , b , c , m_π , the effective gravitational coupling constant α and the cosmological constant Λ . Evidently, the corresponding pattern of solutions is extremely rich. It can be simplified if we note that the rescaling of the parameters of the model (7) allows us to fix $a = b = 1$. Further, without loss of generality, we can take $m_\pi = 1$, so for the regular solutions which approach asymptotically an AdS_4 background, only three parameters c , α and Λ remain.

Properties of the self-gravitating solutions of the general model (6) in the asymptotically flat case has been studied before [28], thus we will extend our consideration to the case of $|\Lambda| \neq 0$.

First, we note that, similar to the case of the usual Einstein-Skyrme theory with a negative cosmological constant [17], the branch of self-gravitating solutions emerges smoothly from the corresponding configuration in the fixed AdS background for any values of c and Λ as α start to increase.

On this branch, in the limiting case of the usual Skyrme model, $c = 0$, for a given α the metric function $N(r)$ develops a minimum at some $r_0 \neq 0$. This minimum moves towards the origin as $|\Lambda|$ increases, see Fig. 1. The depth of the minimum non-monotonically depends on $|\Lambda|$, it becomes least pronounced for $\Lambda = -1$.

In a contrast, in the general model at $c = 1$, the minimum of the metric function $N(r)$ at $r_0 \neq 0$ for a given α exist for relatively small values of $|\Lambda| < 1$. As $|\Lambda|$ becomes bigger, there is a single minimum of this function at the origin. As expected, the soliton core shrinks with increase of $|\Lambda|$ for any values of c .

Investigating the dependence of the solutions on the effective gravitational coupling constant α , we see that both the value of the metric function $\sigma(0)$ at the origin and the minimum

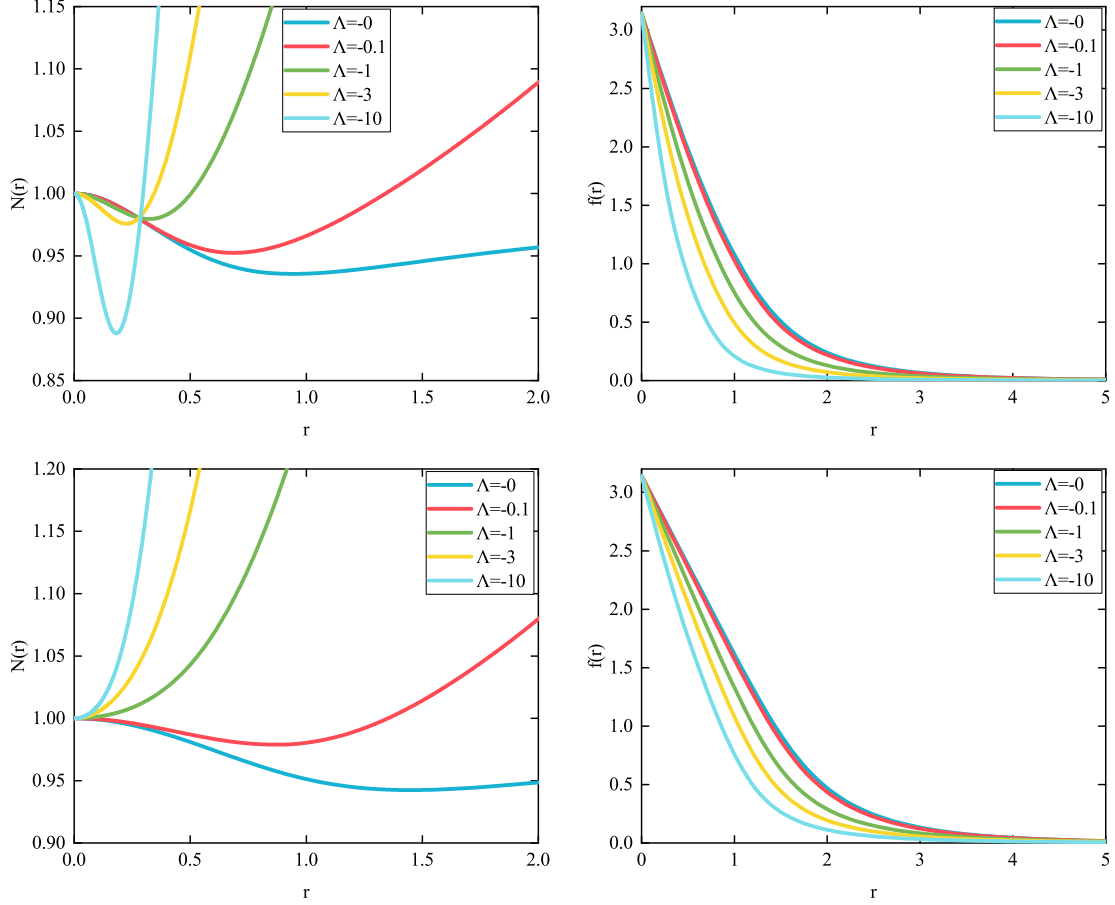


FIG. 1: The metric function $N(r)$ (left column) and the Skyrmion profile function $f(r)$ (right column) of the self-gravitating Skyrmion are plotted as functions of the radial coordinate r for $c = 0$ (upper rows) and $c = 1$ (lower row), for $a = b = 1$, $\alpha = 0.12$, lower branch.

of the metric function $N(r)$, if it exists, monotonically decrease as α increases, as seen in Fig. 2. At the same time, the ADM mass of the solutions monotonically increases. However the rate of decrease of $\sigma(0)$, as well as the rate of increase of the ADM mass, incrementally grows as $|\Lambda|$ is getting large, both for $c = 0$ submodel and for general model with $c = 1$.

This branch extends up to some maximal value of effective gravitational coupling α_{cr} , beyond which the gravitational interaction becomes too strong for gravitating Skyrmons to exist. The value of α_{cr} increases with increasing c and it decreases with increasing Λ , see Fig. 3, our numerical results clearly indicate that the regular solutions of the model (6) exist for any arbitrary large values of $|\Lambda|$.

At $\alpha = \alpha_{cr}$ this branch bifurcates with the second, higher-mass branch of regular solutions of the model (6), which extends backwards, see Fig. 2. Our results confirm the observation that the gravitating solitons of the $c = 0$ usual Einstein-Skyrme exist for all values of the coupling constant $\alpha \rightarrow 0$ [28, 29]. In this limit for any values of Λ the rescaled configuration approaches the lowest Bartnik-McKinnon solution of the $SU(2)$ Einstein-Yang-Mills theory

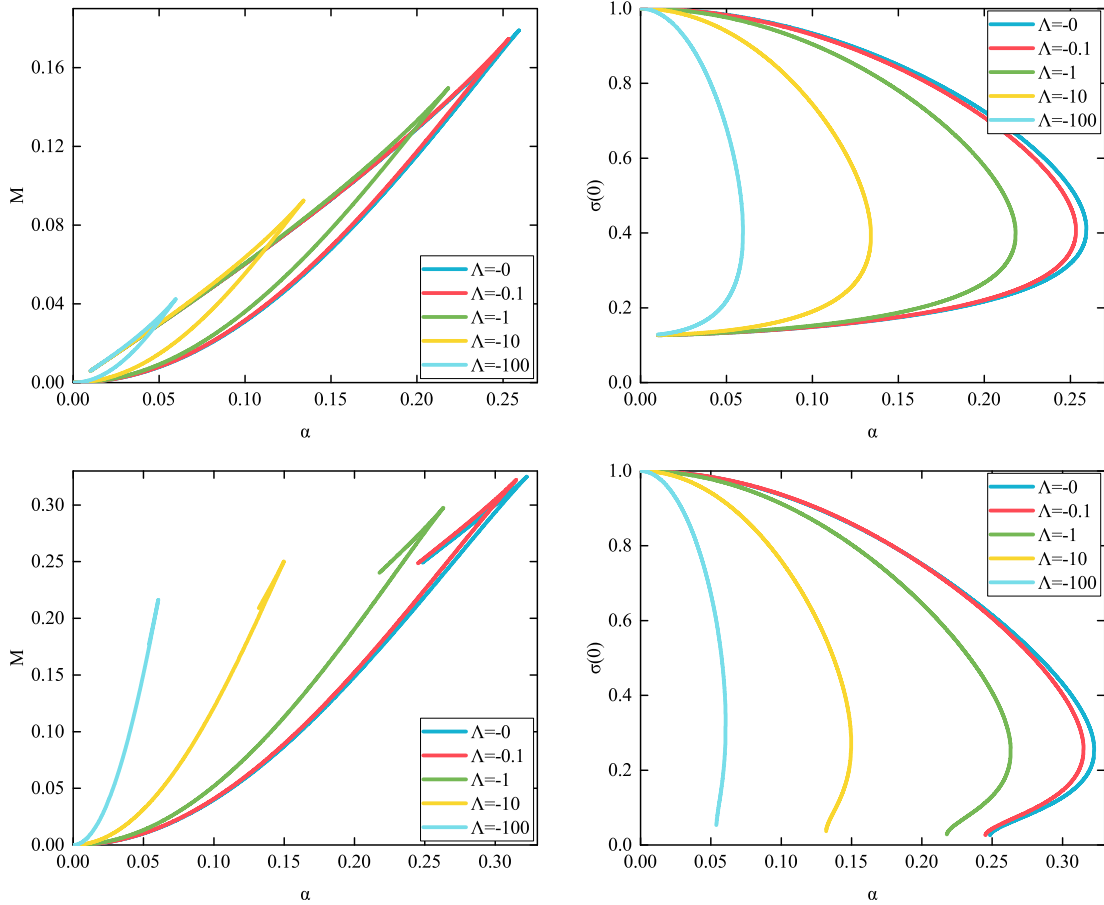


FIG. 2: The value of the metric function σ of the self-gravitating Skyrmsions at the origin, $\sigma(0)$, (left) and the ADM mass of the corresponding solutions (right) are plotted as functions of the coupling constant α for $a = b = m_\pi = 1$ and a few values of Λ . The upper and lower rows correspond to the cases $c = 0$ and $c = 1$, respectively.

[7].

The pattern of evolution of the upper branch regular solutions for non-zero values of the coupling constant c is different since the scaling properties of the general model (6) then are different. We confirm the observation [28] that for any $c \neq 0$ this branch exist only for $\alpha > \alpha_{min}$, at which $\sigma(0) \rightarrow 0$. The value of α_{min} decreases with increasing Λ , cf. Fig. 2. Along this branch the ADM mass of the solutions monotonically decreases, the branch us getting shorter as Λ increases.

We can also consider limiting submodels $\mathcal{L}_0 + \mathcal{L}_4 + \mathcal{L}_6$, $\mathcal{L}_0 + \mathcal{L}_2 + \mathcal{L}_6$ and $\mathcal{L}_0 + \mathcal{L}_6$, for which scaling properties are different, setting the values of the parameters $a = 0$, $b = 0$ and $a = b = 0$ respectively. The case $b = 0$, however, reveals no interesting features since the behavior of the solutions actually follows the pattern we observed in general case. As in the asymptotically flat case, the solutions of the $a = 0$ submodels are solitons with compact support for any values of Λ , the size of the compacton is decreasing as $|\Lambda|$ grows.

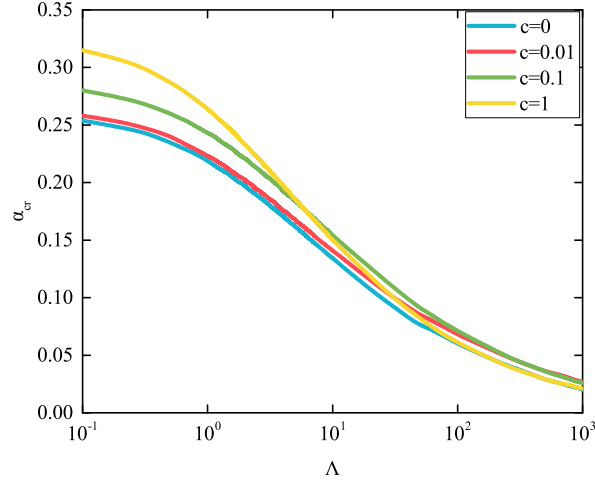


FIG. 3: The critical value of the gravitational coupling α_{cr} is shown as function of $|\Lambda|$ for some set of values of c .

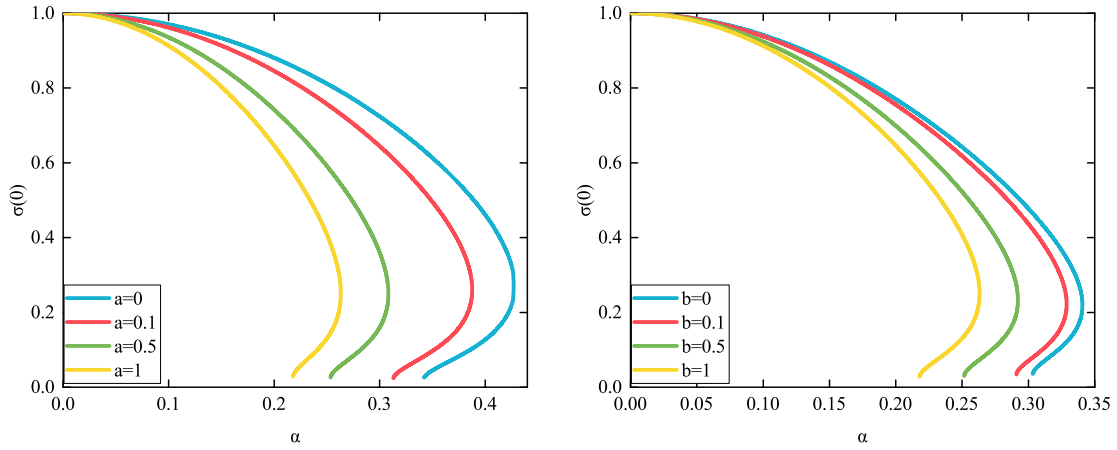


FIG. 4: The value of the metric function $\sigma(0)$ of the self-gravitating Skyrmons at the origin is shown as function of the gravitational coupling constants α for some set of values of the coupling constant a at $b = 1$ (left plot) and for some set of values of the coupling constant b at $a = 1$ (right plot), at $c = 1$ and $\Lambda = 1$.

Qualitatively, the pattern of evolutions of these regular self-gravitating Skyrmons as other parameters of the $a = 0$ submodel are varying, is similar to the general case of the full $a \neq 0$ model, in particular, as both parameters a and b are set to zero, the second, backward branch ceases to exist.

Note that for a fixed value of c , both the value of α_{cr} and the ADM mass of the configuration increase as a decreases. Also the maximal value of the effective gravitational coupling increases as b decreases, see Fig. 4.

IV. HAIRY BLACK HOLES IN THE GENERALIZED EINSTEIN-SKYRME-ADS MODEL

It was pointed out recently that the $b = 0$ submodels without the Skyrme term in the asymptotically flat 3+1 dim spacetime do not possess hairy black hole solutions, even though they support both the static flat space solutions and the regular self-gravitating Skyrmons [28, 29]. In the general model, similar to the usual Einstein-Skyrme $\mathcal{L}_2 + \mathcal{L}_4$ submodel, there are branches of two types, the α -branches which exist at fixed value of the event horizon radius r_h , and r_h -branches for a fixed value of the gravitational coupling α [4, 6]. The black holes with Skyrmonic hair cannot be arbitrarily large, there is a secondary r_h -branch, extending backward towards $r_h = 0$ for sufficiently small, or zero values of c [28, 29]. In other words, the r_h -branch yields a link between the regular solutions on the lower and upper branches at given α .

Let us now consider the black hole solutions with general Skyrmonic hair in asymptotically AdS_4 spacetime. The area of the spherically symmetric horizon is $A = 4\pi r_h^2$ with the entropy $S = A/4$ and the surface gravity of static black hole can be evaluated from the metric functions on the horizon:

$$\kappa^2 = -\frac{1}{4}g^{tt}g^{rr}(\partial_r g_{tt})^2 \quad (16)$$

The Hawking temperature of the static Schwarzschild-like black hole is related to the surface gravity as $T_H = \kappa/(2\pi)$, in the model with cosmological constant Λ without the matter field it is

$$T_H = \frac{1}{4\pi r_h} (1 - \Lambda r_h^2) \quad (17)$$

Considering the black holes with general Skyrme hair, we will imply the spherically metric in the usual AdS -Schwarzschild coordinates (11) with the metric function $N(r)$ related to the mass function $M(r)$ via (12). Then the Hawking temperature of the hairy black hole can be evaluated as

$$T = \frac{1}{4\pi} \sigma(r_h) N'(r_h) \quad (18)$$

As in the case $\Lambda = 0$, the boundary conditions on the matter field and on the metric function must secure the zeroth law of the black holes: the invariants like the surface gravity, and the Kretschmann scalar $K = R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}$ must be finite on the event horizon. In the case under consideration the asymptotic expansion of the fields in powers of $r - r_h$ in the near-horizon area yields

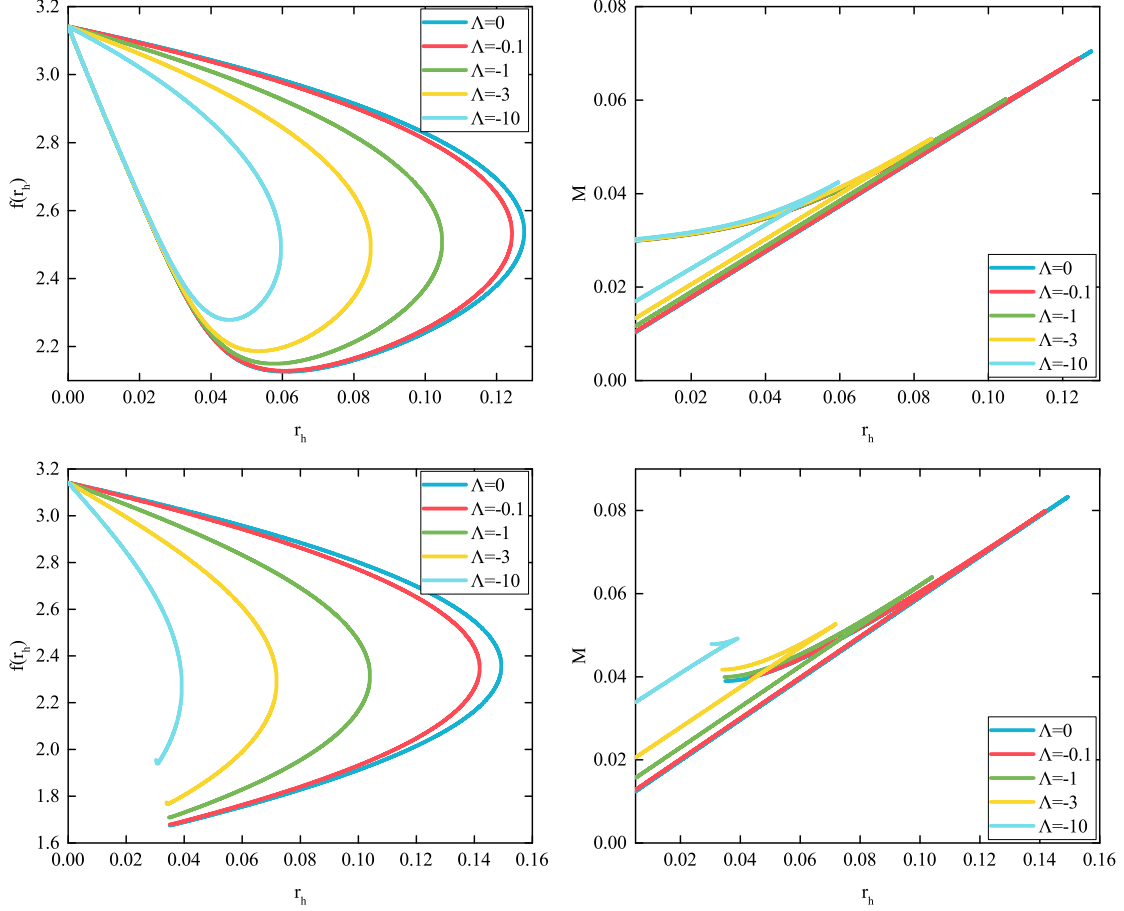


FIG. 5: The value of the Skyrme field profile function $f(r_h)$ at the event horizon (left column) and the ADM mass of the solution (right column) are plotted as functions of the horizon radius r_h for $\alpha = 0.05$, $a = b = m_\pi = 1$, $c = 0$ and $c = 1$ (upper and lower rows, respectively), and some set of values of the cosmological constant Λ .

$$\begin{aligned}
 N &= N_1 (r - r_h) + O(r - r_h)^2, \\
 \sigma &= \sigma_h + \frac{\alpha^2 \sigma_h J^2}{8r_h^3 H N_1^2} (r - r_h) + O(r - r_h)^2, \\
 f &= f_h + \frac{J}{2H N_1} (r - r_h) + O(r - r_h)^2,
 \end{aligned} \tag{19}$$

where where we introduced the shorthand notations

$$\begin{aligned}
 N_1 &= \frac{1}{r_h} - \Lambda r_h - \frac{\alpha^2}{2} \left(m_\pi^2 r_h (1 - \cos f_h) + \frac{2a \sin^2 f_h}{r_h} + \frac{b \sin^4 f_h}{r_h^3} \right), \\
 J &= m_\pi^2 r_h^4 \sin f_h + 4b \sin^3 f_h \cos f_h + 2a r_h^2 \sin 2f_h, \\
 H &= a r_h^4 + 2b r_h^2 \sin^2 f_h + c \sin^4 f_h.
 \end{aligned} \tag{20}$$

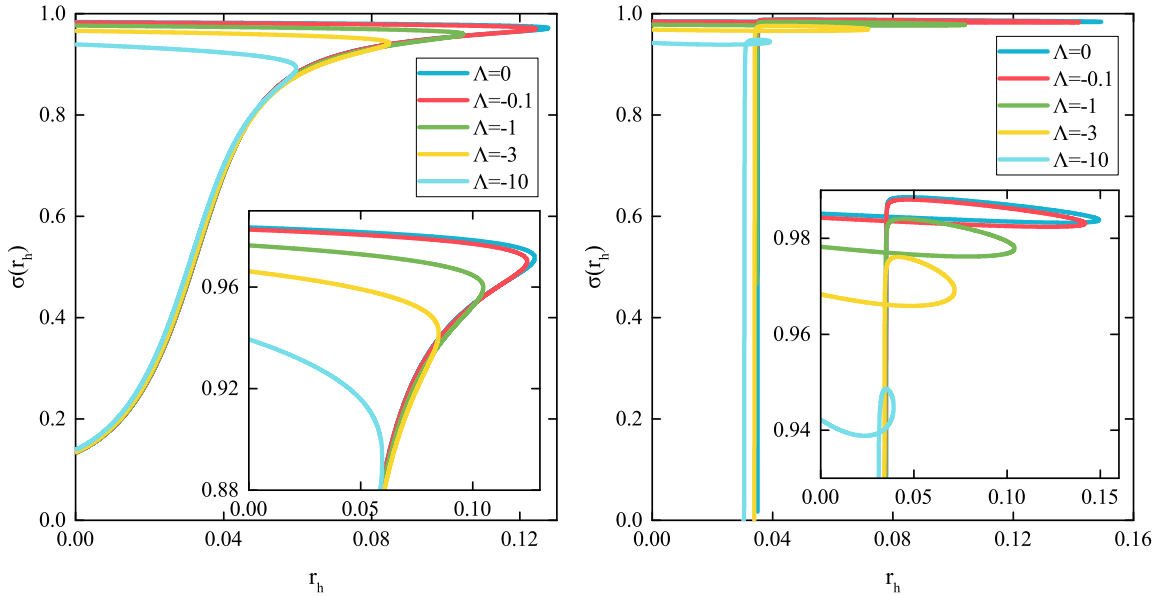


FIG. 6: The values of the metric function $\sigma(r_h)$ at the event horizon of the $c = 0$ submodel (left plot) and the general model at $c = 1$ (right plot) are shown as functions of the horizon radius r_h at $\alpha = 0.05$ and some set of values of the cosmological constant Λ .

Here f_h and σ_h are the values of the Skyrme field profile function and the metric function σ at the event horizon, respectively. Note that it follows from the asymptotic expansion of the metric function $N(r)$ that increasing the value of $|\Lambda|$ yields qualitatively the same effects as increasing the value of the effective gravitational constant α [17].

So, we have to impose the following set of the boundary conditions on the event horizon and on the AdS asymptotic

$$N(r_h) = 0, \quad M'(\infty) = 0, \quad f(\infty) = 0, \quad \sigma(\infty) = 1. \quad (21)$$

The input parameters in the numerical solution of the system of equations (13) are f_h , σ_h , r_h , α and Λ .

A. Numerical results

The properties of the spherically symmetric black holes with general Skyrme hair in the asymptotically flat case were considered before [28, 29], now we can extend the consideration to the asymptotically AdS spacetime. Indeed, we found that all known $\Lambda = 0$ black hole solutions of the general Einstein-Skyrme model possess generalizations with AdS asymptotics. For a fixed nonzero value of the effective coupling α , the lower in energy r_h -branch of solutions smoothly arises from the corresponding regular self-gravitating configuration.

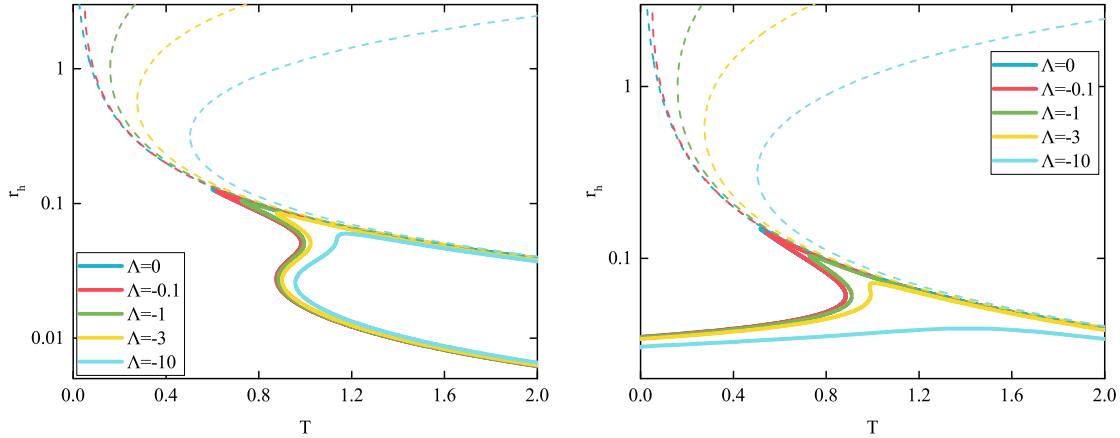


FIG. 7: The Hawking temperature is shown as a function of the event horizon radius for several sets of solutions of $c = 0$ submodel (left) and $c = 1$ general model (right) at different values of Λ for $\alpha = 0.05$, $a = b = m_\pi = 1$. The corresponding temperatures of the Schwarzschild-AdS black holes are plotted as reference curves.

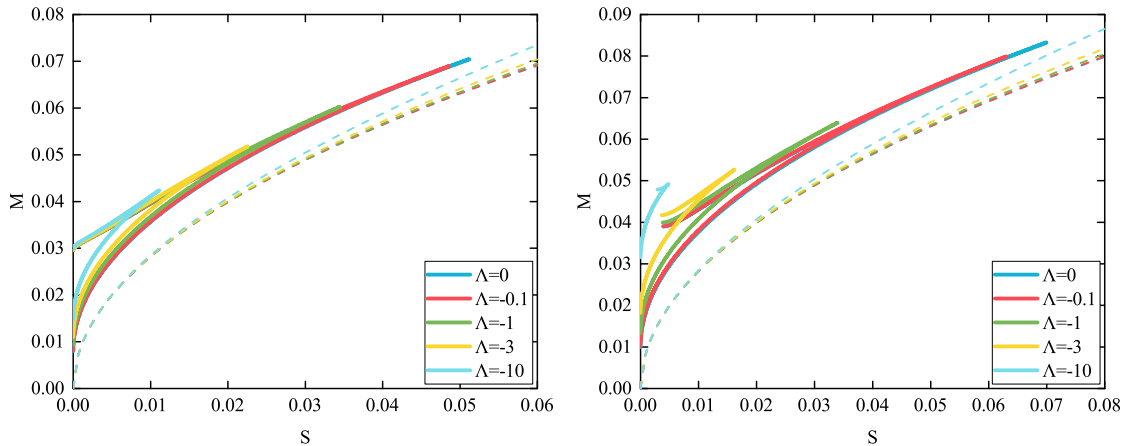


FIG. 8: The mass-entropy curves for the static black holes in the general Einstein-Skyrme model are plotted for several sets of solutions of $c = 0$ submodel (left) and $c = 1$ general model (right) at different values of Λ for $\alpha = 0.05$, $a = b = m_\pi = 1$. The corresponding dependencies of the Schwarzschild-AdS black holes are plotted as reference curves.

This configuration can be viewed as a small AdS-Schwarzschild black hole, embedded into the Skyrmeion [5].

In Fig. 5 we present some results of the numerical integration for two sets of solutions with $c = 0$ and $c = 1$ at $\alpha = 0.05$ and $a = b = m_\pi = 1$. The control parameter of the numerics is r_h , which fixes the size of the event horizon.

We observe that for any values of c , Λ and, for allowed on the lower regular branch value of the effective gravitational constant α , the corresponding branch of black hole solutions

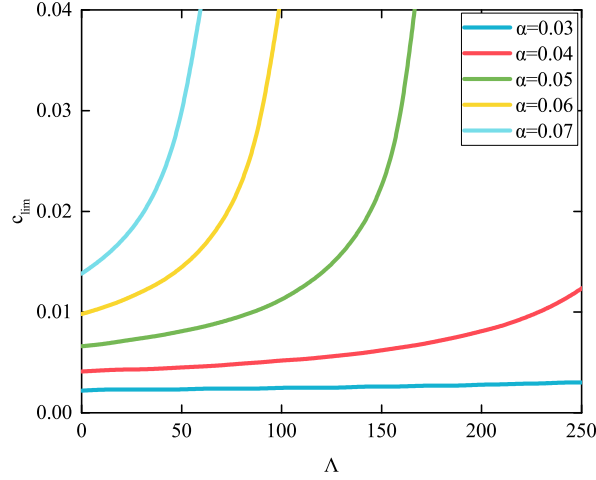


FIG. 9: The critical value of the coupling c , below which the lower and upper branches of the hairy black holes are closed, is shown as a function of the cosmological constant Λ for some set of values of α .

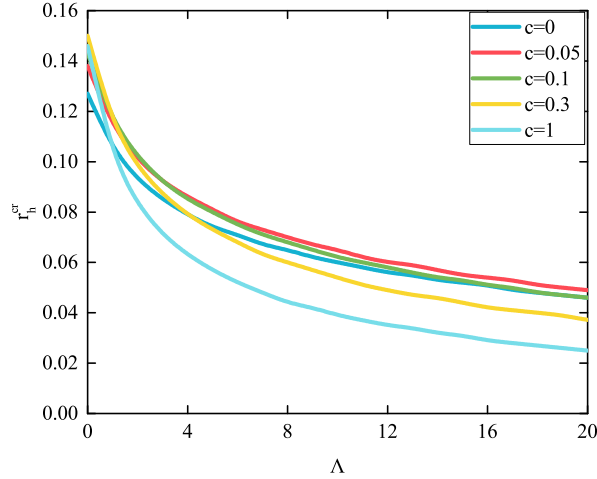


FIG. 10: The critical maximal value of the event horizon radius r_h^{cr} is shown as a function of the cosmological constant Λ for some set of values of c for $\alpha = 0.05$ and $a = b = m_\pi = 1$.

always emerges from the corresponding regular self-gravitating Skymion with a vanishing horizon area $r_h \rightarrow 0$ and a nonzero mass. As expected, in general there is not much difference between the pattern of evolutions of the solutions of the $c = 0$ submodel and the $c = 1$ general model along this lower in mass branch. Both the values of the Skyrme field profile function $f(r_h)$ and the metric function $\sigma(r_h)$ on the event horizon are monotonically decreasing as the horizon radius r_h increases, see Fig. 5. The mass of the configurations is increasing along this branch, see Fig. 5, while the value of the metric function σ_h at the horizon slowly decreases, see Fig. 6. As seen in Fig. 7, on the lower branch the Hawking

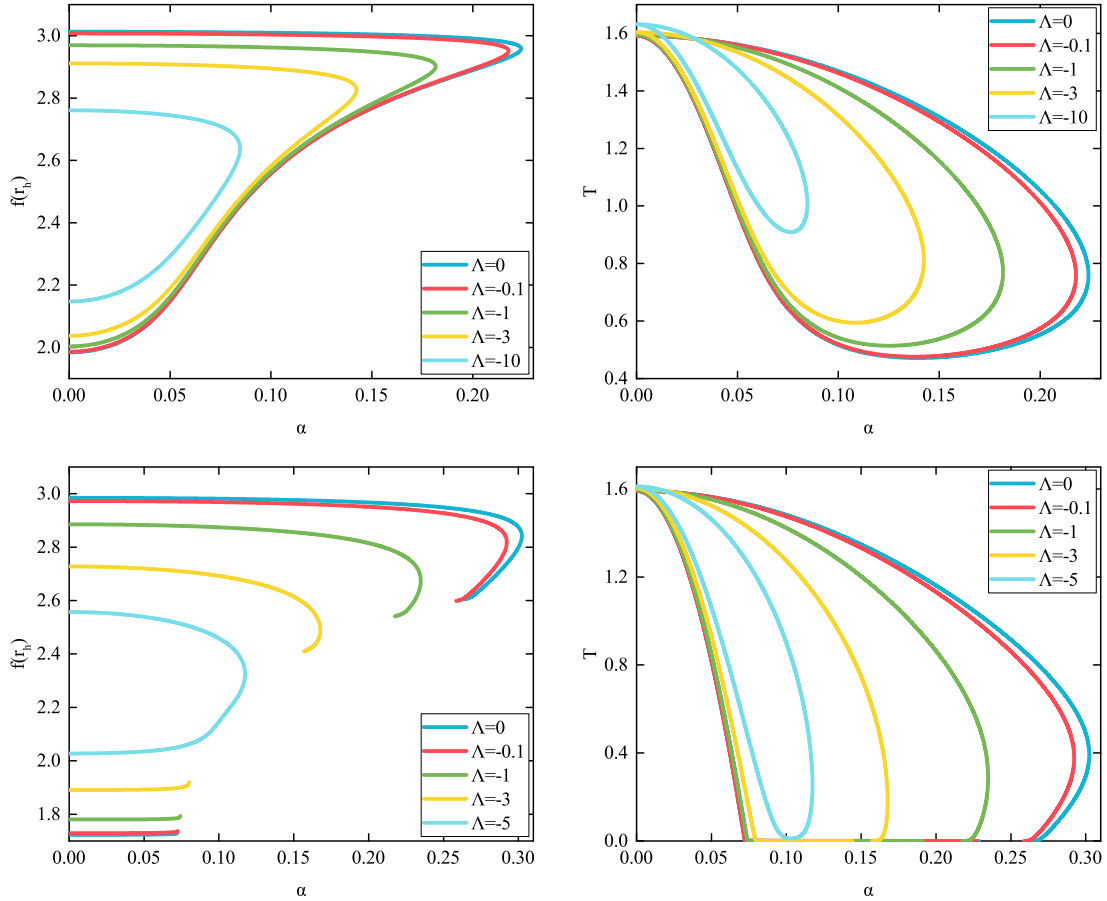


FIG. 11: The values of the Skyrme field profile function at the event horizon, $f(r_h)$, (left column) and the Hawking temperature of the Skyrme black hole (right column) are plotted as functions of the effective gravitational coupling constant α for $c = 0$ (upper row) and $c = 1$ (lower row), $r_h = 0.05$ and some set of values of cosmological constant Λ .

temperature decreases as r_h grows.

The lower branch always terminates at some critical maximal value of the horizon radius r_h^{cr} . There it bifurcates with a secondary, upper (higher-mass) branch, which extends backward in r_h . The upper branch solutions have higher entropy than the solutions on the lower branch.

The curves, which show the dependency of the mass of the configuration on the horizon radius, form a typical cusp at the critical point, as exhibited in Fig. 12, right plots. As expected, the value of the critical horizon radius r_h^{cr} decreases as the absolute value of the cosmological constant Λ grows, see Fig. 10.

Remarkably, the pattern of evolution of the hairy black holes along the upper branch critically depends on the value of the parameter c [28, 29], although the bifurcation into two r_h branches continues to hold for any value of c . In the Einstein-Skyrme submodel with $c = 0$ the upper branch always extends backwards to the limit $r_h = 0$ for any values of Λ

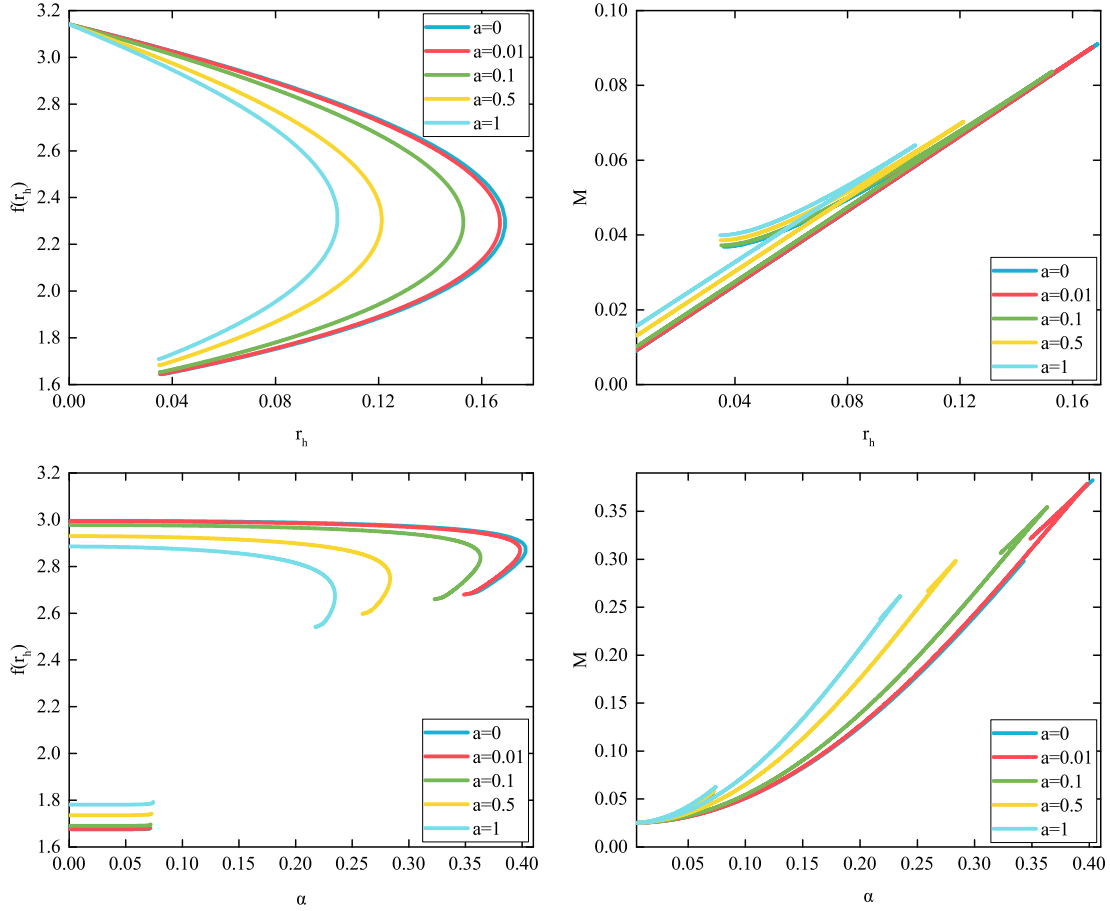


FIG. 12: The value of the Skyrme field profile function at the event horizon, $f(r_h)$, (left column) and the ADM mass of the solution (right column) are plotted as functions of the event horizon radius r_h for $\alpha = 0.05$ (upper row), and the effective gravitational coupling constant α for $r_h = 0.05$ (lower row) for some set of values of a at $b = c = m_\pi = 1$ and $\Lambda = -1$.

and for any α , see Figs. 5,6. As $|\Lambda|$ grows, this branch becomes shorter, however it always approach the limiting $r_h = 0$ solution on the corresponding upper branch of the regular self-gravitating Skyrmons in asymptotically AdS spacetime. This case was considered before in [17], our results nicely agree with observations reported in this paper. As one can see in Figs. 12, along the upper branch the value of the Skyrme field profile function on the event horizon initially continues to decrease as r_h decreases, it approaches some minimal value and then rapidly increases up to $f(r_h) = \pi$ in the limit $r_h \rightarrow 0$. The minimal value of the function $f(r_h)$ slightly increases as $|\Lambda|$ increases. The value of the second metric function on the horizon $\sigma(r_h)$ on the upper branch is rapidly decreasing, as expected, see Fig. 6, left plot.

Interestingly, the Hawking temperature of these hairy black holes exhibits non-monotonical behavior, it starts to increase as r_h decreasing along this branch, then on some subinterval of values of r_h it decreases again, and finally it rapidly grows as r_h continues to

decrease, see Fig. 7, left plot. Thus, for some interval of values of the temperature, there are three solutions with different masses at a given T . For relatively small values of $|\Lambda|$ the absolute minimum of the temperature corresponds to the maximal radius of the horizon r_h^{cr} . As $|\Lambda|$ increases, the second minimum of the temperature becomes the absolute minimum.

As we can see, for a fixed value of r_h the Hawking temperature of the Skyrme black hole decreases as $|\Lambda|$ increases, although for the Schwarzschild-Anti-de Sitter configuration it increases (17). To explain this observation we notice that, as in follows from (18), the temperature is proportional to the product of values of the metric function σ and the derivative of the second metric function $N(r)$ at the event horizon. While σ_h decreases with increasing of Λ , cf. Fig. 6, the derivative of the metric function N increases, see (20). So, the Hawking temperature T_H decreases because σ_h decreases faster than $N'(r_h)$ increases as $|\Lambda|$ grows.

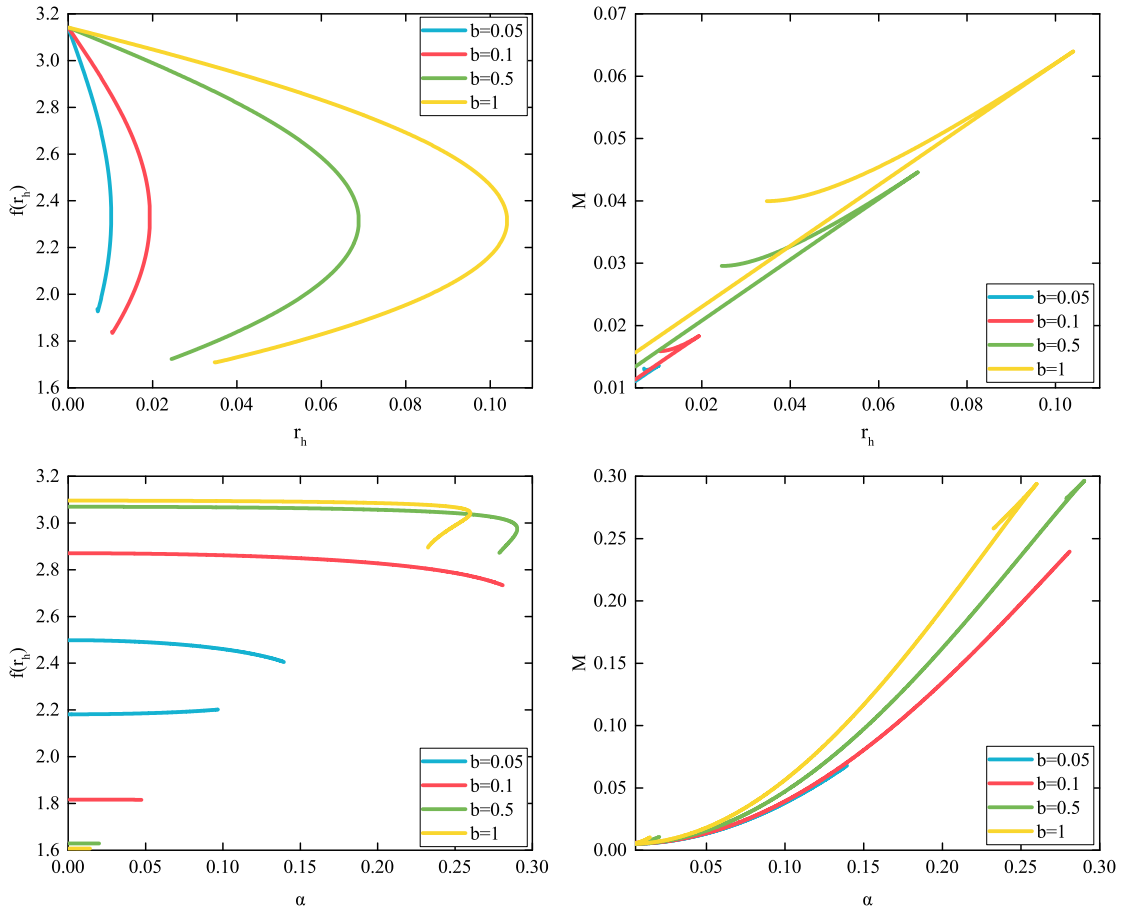


FIG. 13: The value of the Skyrme field profile function at the event horizon, $f(r_h)$, (left column) and the ADM mass of the solutions (right column) as functions of the event horizon radius r_h at $\alpha = 0.05$ (upper row) and the effective gravitational coupling constant α at $r_h = 0.01$ (lower row) for some values of b , $a = c = m_\pi = 1$, and $\Lambda = -1$.

It may be interesting to consider the thermodynamic properties of the hairy black holes in the general Einstein-Skyrme model in AdS spacetime and compare it with the pattern

observed in other related systems, see e.g. [8, 18, 34, 35]. In Fig. 8 we present the mass-entropy relation of the solutions, together with the corresponding curves of the vacuum Schwarzschild-AdS black holes. First, we observe that the two-branch structure exist both for the generalized $c = 1$ Einstein-Skyrme model and for the $c = 0$ submodel. The upper and lower branches described above, correspond to the high- and low-entropy branches, respectively [17]. The lower entropy r_h -branch always originates from the corresponding regular self-gravitating configuration, the mass and the entropy of the hairy black hole are monotonously increasing along this branch. The curve $T(r_h)$ is just below the corresponding lower branch of the Schwarzschild-AdS solution, see Fig. 7.

The entropy and the mass are maximal at the critical point, where the curve has a cusp merging the second, higher entropy branch. This branch is thermodynamically unstable, as in the case of other hairy black holes [8]. The mass and the values of the fields on the event horizon $f(r_h)$, $\sigma(r_h)$ are monotonically decreasing as r_h is decreasing on this branch. As in the case of the asymptotically flat space, for the general $c = 1$ model the upper branch terminates at the limiting (extremal) zero temperature solution, which still possess finite non-zero mass and entropy, see Fig. 8, there is no solutions for values of the r_h smaller than some critical value r_h^{ex} . As $|\Lambda|$ increases, the maximal entropy and the mass of solutions decreases, the branches become shorter. Evidently, the same effect yields increasing of the gravitational coupling α as other parameters of the model remain fixed. The novel feature present in the case of the asymptotically AdS spacetime is that the cosmological constant Λ reduces the entropy.

As we have seen, the temperature of hairy black holes along the upper branch depends on r_h nonmonotonically, it possess a maximum at some point, see Fig. 7. Note that, similar to the case of asymptotically flat spacetime, the limiting behavior of the solution on this branch critically depends on the value of coupling constant c [28, 29]. For $c \leq c_{lim}$, where c_{lim} is some limiting value of c , the temperature, which is initially decreasing along this branch, starts to increase rapidly as $r_h \rightarrow 0$. Thus, the r_h -branches are closed, in the sense that both the lower and the upper branches are linked to the corresponding regular self-gravitating Skyrmons in the limit $r_h = 0$ with infinite Hawking temperature.

In Fig. 15, we exhibit the profile functions of the Skyrmon on the upper and lower branches for a few values of the cosmological constant Λ . Clearly, one can see that the effect of variation of Λ is much more significant for the lower branch configurations.

While increasing the value of the cosmological constant $|\Lambda|$, and/or the effective gravitational coupling α , the critical value c_{lim} rapidly grows, see Fig. 9. Physically, we can expect that for any value of Λ there exists such a value of α , for which both r_h -branches are closed for any value of c within range $0 \leq c \leq 1$.

Our numerical results further indicate that, for $c > c_{lim}$ the solutions on the unstable upper branch approach some finite value at $r_h = r_h^{ex}$, beyond which it cease to exist. At this point both the Hawking temperature of the solution and the value of the metric function σ

at the event horizon, $\sigma(r_h)$, approach zero, see Figs. 7,6.

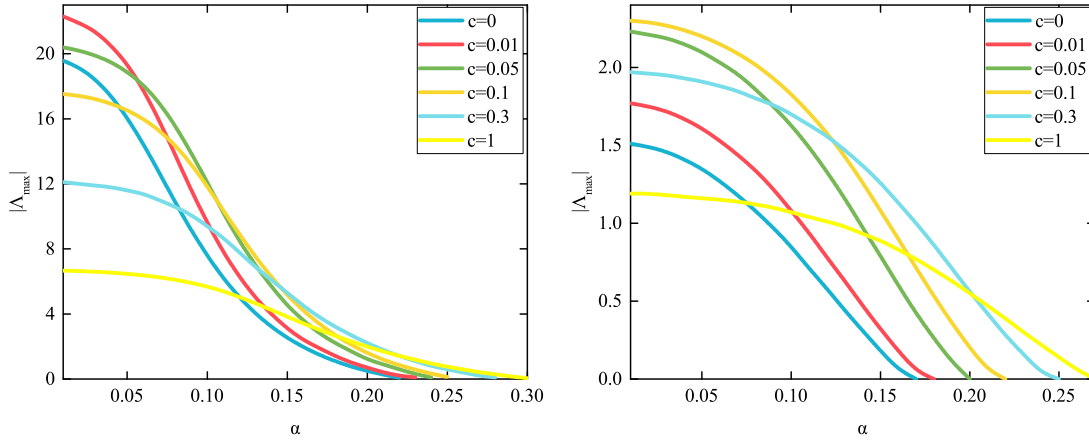


FIG. 14: The maximal value of the cosmological constant Λ_{max} , for which hairy black holes may exist in a general Skyrme model, is plotted as a function of the effective gravitational coupling constant α for $r_h = 0.05$ (left), $r_h = 0.1$ (right) and some set of values of c .

It is worth to point out that the critical values r_h^{cr} and r_h^{ex} typically depend on the coupling constant c and on Λ . Both of them are monotonically decreasing as $|\Lambda|$ increases, while r_h^{ex} monotonically increases with increasing of c . However the form of dependence of r_h^{cr} on c is a bit more complex, with a maximum for some value of r_h^{cr} . In Fig. 10 we present the corresponding maximal values of the event horizon radius as functions of Λ for some set of values of c .

Let us also investigate the dependence of the hairy black hole solutions on the effective gravitational constant α for a fixed value of the horizon radius r_h . Here we also obtain two branches of solutions, linked to the corresponding configurations on the lower and upper r_h branches, respectively. These α -branches bifurcate at some maximal value, say α^{cr} , above which the gravitational interaction becomes too strong for solutions to exist. When r_h and Λ are fixed, the mass of the configuration on the first branch increases as α increases, see Fig. 12.

As expected, the upper α -branch solutions may exist along the entire branch only for $c \leq c_{lim}$. Physically, it corresponds to the link to the limiting configuration, which represents the black hole with Bartnik-McKinnon hairs [36, 37]. As $c > c_{lim}$, there are no solutions for the range of parameters $\alpha_1^{ex} < \alpha < \alpha_2^{ex}$, where at the endpoints α_1^{ex} and α_2^{ex} both the Hawking temperature and the value of the metric function σ at the event horizon, approach zero. While increasing the value of the cosmological constant Λ the numerical analysis shows that the endpoints α_1^{ex} and α_2^{ex} are coming closer and temperature of black hole increases, thus there is a critical value of Λ for which the singular solutions on the second branch cease to exist, see Fig. 11.

Note that for any given values of the horizon radius r_h and the effective gravitational coupling α there is a maximal value of the cosmological constant Λ , for which the solutions

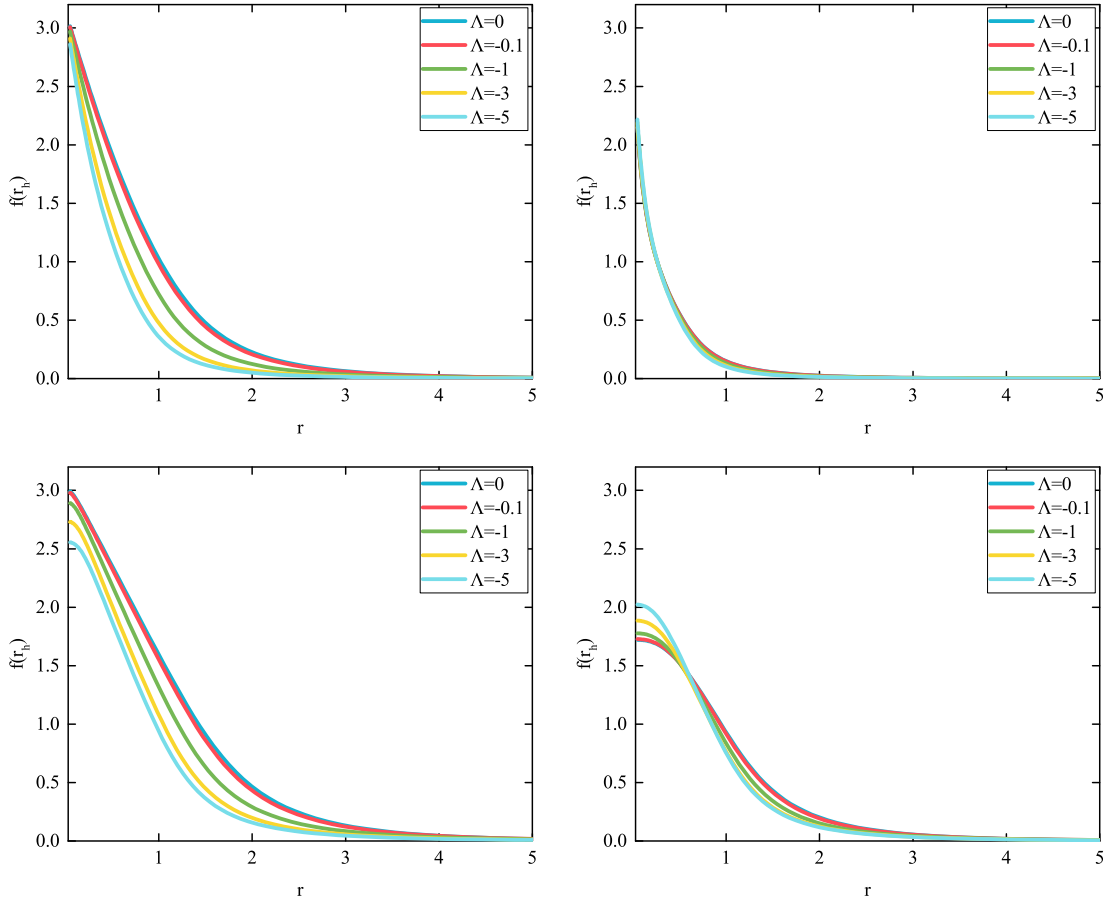


FIG. 15: The Skyrme profile functions $f(r)$ is plotted for several sets of values of cosmological constant Λ at $\alpha = r_h = 0.05$ and $a = b = m_\pi = 1$. The left and right columns show the solutions on the lower and upper branches solutions, respectively. The upper row corresponds to the $c = 0$ Einstein-Skyrme submodel, the lower row corresponds to the general $c = 1$ model.

may exist. This value monotonically decreases with increasing of α , however its dependency on the coupling constant c is not monotonical, it possess a maximum at some value of this parameter, as illustrated in Fig. 14. We also recall that the solutions may exist only for values of α less than some maximal value α_{cr} , which decreases with increasing r_h . Again, with increase of Λ the critical value of α is decreasing.

It might also be interesting to consider the limiting cases $a = 0$, $b = 0$ and $a = b = 0$, for which the scaling properties of the corresponding reduced Einstein-Skyrme submodels are different. As we know, in the asymptotically flat spacetime the role of the Skyrme term \mathcal{L}_4 is special, both the submodel $\mathcal{L}_0 + \mathcal{L}_2 + \mathcal{L}_6$ and the BPS submodel $\mathcal{L}_0 + \mathcal{L}_6$ do not support hairy black holes [28, 29]. On the other hand, the solutions of the reduced $\mathcal{L}_0 + \mathcal{L}_4 + \mathcal{L}_6$ submodel represent a new type the black holes with compact hair.

Extending our consideration to the case of the asymptotically AdS spacetime, we can see that both the maximal value of event horizon radius r_h^{cr} and the ADM mass of the solutions

rapidly decrease as the parameter b , which multiplies the Skyrme term, is decreasing, see Fig. 13. We can conclude that in the limit $b \rightarrow 0$ there are no hairy black hole solutions for any value of the cosmological constant Λ . We note that, for a given b , the increase of the absolute value of Λ decreases the maximal value of the horizon radius r_h^{cr} .

Finally, we investigate the behavior of the solutions as the parameter a , which multiplies the quadratic term \mathcal{L}_2 , decreases. In Fig. 12 we demonstrate that the maximal value of event horizon radius r_h^{cr} and ADM mass increase as a is decreasing, so we may conclude that the hairy black holes with Skyrme hair persist in the limiting case $a = 0$ for any value of Λ . However, as in the asymptotically flat spacetime, these configurations possess compact hair. Construction of the corresponding solutions remains currently still a numerical challenge.

We note that the role of the Skyrme term remains very special for any values of Λ . Indeed, as both parameters a and b decrease simultaneously, the value of r_h^{cr} is also decreasing and approaches zero. Thus, the pattern we observed for $a = b \rightarrow 0$ is similar to the case of the $b = 0$ submodel, the hairy black holes do not exist in the models without the Skyrme term.

V. CONCLUSIONS

The main purpose of this work was to extend the discussion of the static gravitating Skyrmions and black holes in the generalized Einstein-Skyrme model to the case of theory with a negative cosmological constant. The effect of introduction of the cosmological constant can be summarise as follows: the increase of $|\Lambda|$ qualitatively yields the same effects as increasing the value of the effective gravitational coupling α , in particular the critical value of the horizon radius r_h^{cr} is decreasing as $|\Lambda|$ increase. Not entirely surprisingly, it turns out that the hairy black holes exist for any Λ only if the model contains the Skyrme term \mathcal{L}_4 . Perhaps the most unusual feature of the model considered in this work is the existence of extremal (zero-temperature) limiting black hole solutions for some set of values of the parameters. We show that increase of $|\Lambda|$ may eliminate these singular solutions.

We have focussed on the spherically symmetric Skyrmions of topological degree one, which represent only the simplest possible type of solutions. As a direction for future work, it would be interesting to study the higher charge solutions, which have very geometrical shapes [38]. As a first step towards investigation of the higher degree solutions of the generalised Einstein-Skyrme models one can consider the axially-symmetric charge two Skyrmions [9], or sphaleron solutions of the Einstein-Skyrme model [12].

Clearly, it would be interesting to investigate the spinning Kerr black holes with Skyrme hair. The regular self-gravitating spinning Skyrmions were considered in Ref. [11], however almost nothing is known about the stationary rotating Skyrme black holes which should emerge from the globally regular rotating Skyrmions. One can conjecture that a new type of hair may appear for submodels which do not have such solutions in the static limit, similar to the situation, considered in [39]. It might also be interesting to study how the

(non)existence of hairy Skyrmonic black holes could be influenced by some modification of the gravity like in higher curvature models or in dilaton-Gauss-Bonnet gravity [40].

While in the asymptotically flat space the lower branch solutions are stable and the upper branch solutions are unstable [6, 8], the stability analysis of the static solutions in asymptotically AdS is not obvious. In particular, it was shown that the cosmological constant stabilizes the black hole Skyrmon in the $c = 0$ submodel [17]. Thus, we conjecture that the same would be expected of the solutions of the generalised model. We leave this consideration for future research.

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- [41] Our conventions are slightly different from usual choice, see e.g. [17], the matter field sector differs from the standard one by a factor of one half. Evidently, corresponding rescaling of the cosmological constant Λ allows us to recover the latter conventions.